B.Sc. Semester-V Examination, 2022-23

MATHEMATICS [Programme]

Course ID : 52118 Course Code : SP/MTH/501/DSE-1A

Course Title : Theory of Equations

OR

Linear Programming

Time : 2 Hours Full Marks : 40

The figures in the right-hand margin indicate marks.

Candidates are required to give their answers in their own words as far as practicable.

Notations and synbols have their usual meaning.

(Theory of Equations)

1. Answer any **five** questions: 2×5=10

a) Apply Descartes' rule of signs to determine the total number of real roots of the equation $x^n - 1 = 0$, when n is odd.

b) If $x^3 + 3px + q$ has a factor of the form $(x-a)^2$, then show that $q^2 + 4p^3 = 0$.

c) Form the cubic equation whose two roots are $5,\ 2+\sqrt{7}$.

d) Find the condition that the roots of the equation $x^3 - px + qx - r = 0$ are in geometric progression.

e) Find the quotient and remainder when $x^4 - 1 = 0$ is divided by $x - 2$.

f) Remove the second-degree term from $x^3 - 15x^2 - 33x + 847 = 0$ using translation.

g) Find the equation whose roots are reciprocal to the roots of $x^3 + 2x^2 - 2 = 0$.

h) Calculate the first Sturm's function of the equation $x^4 + 4x^3 - x^2 - 2x - 5 = 0$.

2. Answer any **four** questions: 5×4=20

a) i) Find the remainder when $x^{10} + x^7 + x^4 + x^3 + 1$ is divisible by $x^2 + 1$.

ii) If $x^2 + px + 1$ is a factor of $ax^3 + bx + c$, then prove that $a^2 - c^2 = ab$. 3+2

b) Solve the cubic equation $x^3 - 9x + 28 = 0$ by Cardon's method.

c) Find the equation each of whose roots is greater by 2 than a root of the equation $x^3 - 5x^2 + 6x - 3 = 0$.

d) i) If α, β, γ is the roots of the equation $x^3 + px + q = 0$, then find the equation whose roots are $\frac{\alpha}{\beta}, \frac{\beta}{\alpha}, \frac{\beta}{\gamma}, \frac{\gamma}{\beta}, \frac{\gamma}{\alpha}, \frac{\alpha}{\gamma}$.

ii) State Rolle's theorem for algebraic equation. 4+1

e) i) Find the special roots of the equation $x^6 = 1$.

ii) Show that the equation $x^3 - 3x^2 - 9x + 27 = 0$ has a multiple root. 2+3

f) If α be an imaginary root of $x^n - 1 = 0$, where n is a prime number, prove that $(1-\alpha)(1-\alpha^2)...(1-\alpha^{n-1}) = n$.

3. Answer any **one** question: 10×1=10

a) i) Show that the equation

$$x^4 + 4x^3 - 2x^2 - 12x = 0$$

has four real and unequal roots.

ii) Solve

$$x^4 + 2x^3 - 16x^2 - 22x + 7 = 0$$

considering $2 - \sqrt{3}$ as its one root.

iii) Let α, β, γ be the roots of the equation $x^3 + qx + r = 0$, then find the value of $\sum \frac{1}{\alpha^2 - \beta\gamma}$. 4+3+3

b) i) Obtain the upper and lower limit of the positive roots of $f(x)=0$, where $f(x) = x^5 - 4x^3 + x^2 - x + 1$.

ii) Show that the equation

$$x^n - nqx + (n-1)r = 0$$

will have a pair of equal roots if $q^n = r^{n-1}$.

iii) Solve the equation

$$x^4 - 7x^3 + 14x^2 - 2x - 12 = 0,$$

one root being $(1+\sqrt{3})$. 3+3+4

(Linear Programming)

1. Answer any **five** questions: 2×5=10

a) Define convex set with example.

b) By using artificial variables (In case of solving the LPP by Big M-Method) convert the following LPP in standard form only.

Maximize $z = 4x_1 - 3x_2 + x_3$

subject to $x_1 + x_2 + 2x_3 = 5$

$2x_1 - 3x_2 + x_3 = 2$

$x_1, x_2, x_3 \geq 0$.

c) Prove that a hyperplane is a convex set.

d) Show that $x_1 = 5$, $x_2 = 0$, $x_3 = -1$ is a basic solution of the system of equations

$$x_1 + 2x_2 + x_3 = 4,\ 2x_1 + x_2 + 5x_3 = 5$$

e) Find the dual of the following linear programming problem:

Minimize $z = -6x_1 - 8x_2 + 10x_3$

subject to $x_1 + x_2 - x_3 \geq 2$

$2x_1 - x_3 \geq 1$

$x_1, x_2, x_3 \geq 0.$

f) What do you mean by a balanced transportation problem?

g) Obtain an initial basic feasible solution to the following transportation problem using the North-West Corner method:

	DESTINATION			a_i
ORIGINS	5	1	8	12
	2	4	0	14
	3	6	7	4
b_j	9	10	11	

h) Solve the following game problem and find the value of the game.

3	−2
−2	3

2. Answer any **four** questions: 5×4=20

a) Following is the starting tableau of an LPP by the simplex method (for a maximization problem), in an incomplete form

			c_j						
C_B	B	X_B	b	a_1	a_2	a_3	a_4	a_5	a_6
0	a_4	x_4	2	2	1	1			
0	a_5	x_5	5	1	2	3			
0	a_6	x_6	6	2	2	1			
	$(z_j - c_j)$			-3	-1	-3	0	0	0

i) Complete the objective row and the tableau.

ii) Write down the LPP in its standard form from the tableau.

iii) Write down the actual problem.

iv) Find the departing and the entering vectors and write down the next tableau.

1+1+1+2

b) i) Solve the LPP by graphical method

Maximize $z = x_1 + x_2$

subject to $5x_1 + 10x_2 \leq 50$

$x_1 + x_2 \geq 1$

$x_2 \leq 4$

$x_1, x_2 \geq 0$ 3

ii) Determine the position of the points (–6, 1, 7, 2) and (1, 2, –4, 1) with respect to the hyperplane $3x_1 + 2x_2 + 4x_3 + 7x_4 = 8$.

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c) Solve by "Big M-method":

Maximize $z = 4x_1 + 2x_2$

subject to $2x_1 + x_2 \leq 4$

$5x_1 + 3x_2 \geq 15$

$x_1, x_2 \geq 0$

d) Give the dual of the following linear programming problem and hence solve it

Maximize $z = 3x_1 - 2x_2$

subject to $x_1 \leq 4$

$x_2 \leq 6$

$x_1 + x_2 \leq 5$

$-x_2 \leq -1$

$x_1, x_2 \geq 0$

e) Find the minimum cost solution for the 4×4 assignment problem whose cost coefficient are as given below

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{cccc} I & II & III & IV \end{array} \\ \begin{bmatrix} 4 & 5 & 3 & 2 \\ 1 & 4 & -2 & 3 \\ 4 & 2 & 1 & -5 \end{bmatrix} \end{array}$$

f) Show that, for the game problem

a	0
0	b

The optimal strategies are $\left(\frac{b}{a+b}, \frac{a}{a+b}\right)$ and

$\frac{ab}{a+b}$ is value of the game.

3. Answer any **one** question: 10×1=10

a) i) Determine the convex hull of the points (0, 0), (0, 1), (1, 2), (1, 1), (4, 0).

ii) Solve by simplex method:

Maximize $z = 4x_1 + 14x_2$

subject to $2x_1 + 7x_2 \le 21$

$7x_1 + 2x_2 \le 21$

$x_1, x_2 \ge 0$

Will the above LPP admit an alternative solution? Give reason. If yes, then find another solution of the above LPP.

2+(5+3)=10

b) i) For what value of λ, the game with the following pay off matrix is strictly determinable 5

B

	λ	6	2
A	-1	λ	
	-2	4	λ

ii) For the following pay-off table, transform the zero sum game into equivalent linear programming problem and solve it by simplex method. 5

Player Q

		Q_1	Q_2	Q_3
Player P	P_1	9	1	4
	P_2	0	6	3
	P_3	5	2	8

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